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FLAT RIGID DIE UNDER A TRANSVERSE LOAD

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Different problems relating to the dynamics of mechanisms and structures secured to the surface of the ground require long-term study of the processes which take place in soil under surface loads. One of the simplest and most commonly used means of attachment is the embedded structural element, which for the sake of brevity we will henceforth refer to as a die. The behavior of the die when subjected to normal loads has been studied fairly extensively. At the same time, we know of no studies which have analyzed the problem when a transverse load is applied to a die embedded in soil.

It is natural that the solution of such a problem would require consideration of the irreversible strains undergone by the soil, i.e., require the use of a mathematical model of plasticity for the soil. We propose to use an established method of solving problems of plasticity theory which is based on one of the theorems of limit analysis — the theorem of the upper bound of the limit load. This method is relatively simple and makes it possible to obtain a quantitative solution when the Saint-Venant model (incompressible plastic-rigid medium) is used [1]. The essence of the method is the use of the principal energy equality (equilibrium equations in the integral Lagrangian form)

$$\int_S \sigma_n v dS = \tau_s \int_V H^k dV + \sum_{n=1}^{N} \tau_s \int_{S_n} |[v_\tau^k]| \, dS_n - \int_V X v^k dV. \tag{1}$$

Here, σ_n is the vector of the forces on the surface S; v is the velocity vector; τ_s is the plastic limit; H is the shear rate; $[v_\tau]$ is the jump of the shear component of velocity on the possible surface of discontinuity of velocity S_n; X are the body forces within the volume V.

A key aspect of the use of Eq. (1) is assignment of the kinematically possible velocity fields (these quantities are denoted by the superscript k in (1), which will henceforth be omitted). The specified velocity field makes it possible to calculate the unknown vector of the surface forces σ_n, the above theorem then being used to estimate the upper bound of the actual unit load. It is obvious that the more accurately the flow field is assigned, the more accurate will be the limit load calculated from (1). Two questions arise in connection with this: 1) how should the velocity field be assigned? 2) how should the results of the solution be interpreted? It is best if we put off discussion of the second question until later (after the solution has been obtained). The answer to the first question can be sought by modeling the flow field in the following simple experiment.

We performed the modeling by using a box with a transparent front wall. The flat die put in the box was covered with bank sand, which was emplaced in layers (the dark bands in Fig. 1 represent colored sand). A transverse force was applied to the top part of the die after the die and sand were in place. Visible in Figs. 2 and 3 are the different stages of flow of the soil in the neighborhood of the die. Two main conclusions can be drawn from the figures: 1) a "hinge" is formed, this hinge turning together with the die as a rigid whole (rotation of the die in the opposite direction results in a corresponding rotation of the hinge); 2) the hinge ceases to function with further rotation of the die, and an isolated line appears on the top surface. The soil (sand) is lifted upward along this line. This stage is referred to as the "shovel" stage.

It would be expedient to use the velocity fields corresponding to events that occurred in the box in the theoretical solution of the problem (rather than some hypothetical field).

Hinge Stage. The deformation region is divided into three zones. In polar coordinates r, θ, the velocity field is assigned in the form $u = v_r$, $v = v_\theta$ (Fig. 4)

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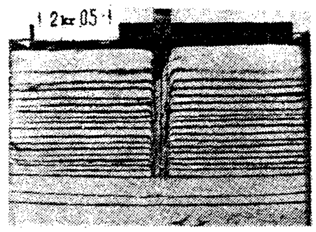

Fig. 1

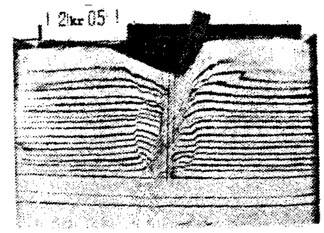

Fig. 2

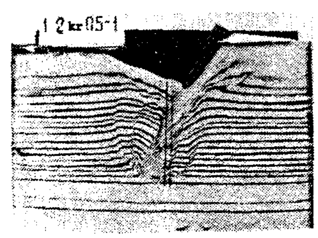

Fig. 3

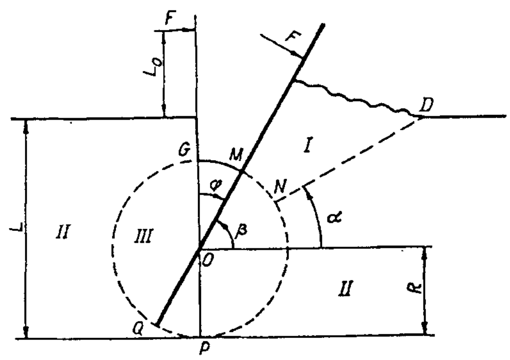

Fig. 4

Zone I is the region of plastic deformation:

$$u = \frac{v_0}{2}\frac{r}{L - R}\frac{1}{\beta - \alpha}\left(1 - \frac{R^2}{r^2}\right), \quad v = -v_0 \frac{r}{L - R}\frac{\theta - \alpha}{\beta - \alpha}.$$

Zone II is the rigid stationary region:

$$u = v = 0.$$

Zone III is the rotating hinge:

$$u = 0, \quad v = -v_0 \frac{r}{L - R},$$

where L is the depth to which the die is embedded; R is the radius of the hinge; α is the angle specifying the boundary of zone I; β is the angle of inclination of the die to the horizon; v_0 is the component of velocity on the die (the velocity of point B).

It is not hard to show that the supposed velocity field is kinematically possible, since it satisfies the incompressibility equation

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta} = 0$$

and the continuity condition on any surface S with the normal **n**

$$[\mathbf{v}\cdot\mathbf{n}]_S = 0.$$

It is evident that the given field is discontinuous — the surfaces S_1 = ND, S_2 = MN, S_3 = NPQG are surfaces of discontinuity of the shear components of velocity.

In zone I

$$H = \frac{v_0}{(L - R)(\beta - \alpha)} \left(1 + \frac{R^2}{r^2}\right),$$

in zones II and III

$$H = 0,$$

so that

$$\int_V H dV = \frac{v_0}{(L-R)(\beta-\alpha)} \left[\frac{(L-R)^2}{2}(\operatorname{ctg}\alpha - \operatorname{ctg}\beta) - R^2 J - R^2(\beta-\alpha)\left(\frac{1}{2} - \ln\frac{L-R}{R}\right)\right],$$

where

$$J = \int_\alpha^\beta \ln \sin\theta d\theta.$$

The sum of the integrals over the surfaces of velocity discontinuity is

$$\sum_{n=1}^{3} \int_{S_n} |[v_\tau]| \, dS_n = \frac{v_0}{L-R}\left[\frac{R^2}{2}(3\pi + \beta + \alpha) + \right.$$
$$\left. + \frac{L^2 + R^2\cos^2\alpha - 2LR}{4(\beta-\alpha)\sin^2\alpha} - \frac{R^2}{2(\beta-\alpha)} \ln\frac{L-R}{R\sin\alpha}\right].$$

The integral of the body forces (here, the force of gravity)

$$\int_V \mathrm{X}\mathrm{v} dV = -\rho g \int_V (u \sin\theta + v\cos\theta)\, dV = -\rho g v_0 \left[\frac{(L-R)^2}{6\sin^2\beta} + \frac{R^3 \sin\beta}{3(L-R)} - \frac{R^2}{2}\right].$$

The die is assumed to be smooth (i.e., $\sigma_{nr} = 0$ on the surface of the die), so that the integral in the left side of (1) can be represented in the form

$$\int_S \sigma_n \mathrm{v} dS = \int_S \sigma_{n\theta} v dS = -\frac{v_0}{L-R}\int_S \sigma_{n\theta} r dS.$$

It follows from the condition of equality of the principal moment that the last integral is related as follows to the applied force F:

$$F(L + L_0 - R) + \int_S \sigma_{n\theta} r dS = 0$$

(where L_0 is the coordinate of application of the force F). It follows from this that

$$F = \frac{1}{v_0}\frac{L-R}{L+L_0-R}\int_S \sigma_n \mathrm{v} dS$$

or, with allowance for all of the formulas

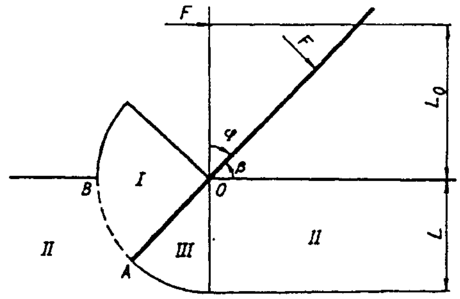

Fig. 5

$$F = \frac{\tau_s R^2}{L + L_0 - R}\left[\frac{2\ln\sin\alpha - 1 - 4J}{4(\beta - \alpha)} - \left(\frac{1}{2(\beta - \alpha)} - 1\right)\ln\frac{L - R}{R} + \right.$$
$$\left. + \frac{\beta + \alpha}{2} + \frac{3\pi}{2} - \frac{1}{2}\right] + \frac{\tau_s (L - R)^2}{2(\beta - \alpha)(L + L_0 - R)}\left[\operatorname{ctg}\alpha - \operatorname{ctg}\beta + \frac{1}{2\sin^2\alpha}\right] + \tag{2}$$
$$+ \rho g \frac{L - R}{L + L_0 - R}\left[\frac{(L - R)^2}{6\sin^2\beta} + \frac{R^3 \sin\beta}{3(L - R)} - \frac{R^2}{2}\right].$$

Formula (2) makes it possible to calculate the drag F with assigned L, L_0, and β; here, R and α are free parameters, i.e., the solution we obtain is a two-parameter solution. Minimizing F with respect to $\alpha \subset (0, \beta)$ and $R \subset (0, L/2)$, we can obtain the upper bound of the limit load in this class of solutions:

$$F_h(\beta) = \min_{\alpha, R} F(\alpha, \beta, R).$$

Let us proceed to description of the second stage of deformation — the "shovel" stage (Fig. 5). At this stage, the die rotates about the point O. The region of deformation is broken down into three zones:

zone I — rotation as a rigid whole about the point O:

$$u = 0, \quad v = -v_A \frac{r}{L}$$

(v_A is the velocity of point A):

zone II — the rigid zone;

$$u = v = 0;$$

zone III — an empty zone without soil. In this case, in all of the zones

$$H = 0.$$

Line AB is the surface of discontinuity of the velocity field;

$$\int_{AB} |[v_\tau]|\, dS = v_A L \beta.$$

The integral of the body force

$$\int_V \mathrm{X} v dV = -\rho g v_A L^2 \frac{\sqrt{2}}{3} \sin\left(\frac{\pi}{4} + \beta\right).$$

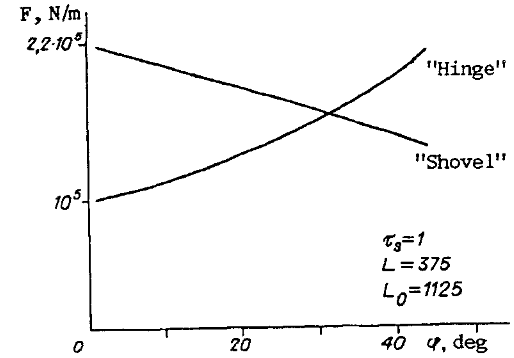

Fig. 6

We represent the integral in the left side of (1) in the form

$$\int_S \sigma_n v dS = -\frac{v_A}{L}\int_S \sigma_{n\theta} r dr,$$

where the last integral is simply connected with the force F from the condition of triviality of the principal moment

$$FL_0 + \int_S \sigma_{n\theta} r dr = 0.$$

The above formulas make it possible to write a simple expression connecting the force F at the "shovel" stage with the angle β:

$$F_{sh} = \frac{L^2}{L_0}\left[\tau_s\beta + \frac{\sqrt{2}}{3}\rho g L \sin\left(\frac{\pi}{4} + \beta\right)\right].$$

The further construction of the solution is obvious.

Since $F_h(\beta)$ and $F_{sh}(\beta)$ are the upper bounds for the actual limit load, for each β we should take

$$F_0(\beta) = \min\{F_h(\beta),\ F_{sh}(\beta)\}.$$

One variant of calculation is shown in Fig. 6. The calculation was performed for the following numerical values: plastic limit $\tau_s = 10^5$ Pa; depth of the die L = 375 cm; distance from the point of application of the transverse force to the free surface $L_0 = 1125$ cm; density of the soil $\rho = 2$ g/cm^3.

It is evident from the figure that up to the critical angle $\varphi_* = \pi/2 - \beta_*$ (where φ is the angle of deviation of the die from the vertical) the load is determined by the field of the hinge, while at $\varphi > \varphi_*$ it is determined by the field of the shovel. Thus, the above method of calculation makes it possible to find the limit load for any angle β (or φ). This is the load at which plastic deformation of the soil begins in the neighborhood of the loaded die. Comparison of the numerical results with the experimental data demonstrates the suitability of the proposed algorithm for engineering calculations.

We believe that the phenomenon of hinged rotation of the soil that we discovered here must be taken into account when the limit load of embedded structural elements is being calculated.

REFERENCES

1. L. M. Kachanov, Principles of the Theory of Plasticity [in Russian], Nauka, Moscow (1969).